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- 2- "Optimizing Hull Steel Weight for Overall Economic Transportation", Marine Week, May 2, (UK-1975), Shama, M. A.,
- 3- "The Cost of Irrationality in Ship Structural Design", PRADS. Int. Conference on Practical Design in Shipbuilding, SNAJ, Tokyo Oct. (Japan-1977), Shama, M. A.,
- 4- "Computer Design of Ships", Bull. Collage of Engineering, Basra University, (Iraq-1977), Shama, M. A.,
- 5- " Economical Consequences of Irrational Structural Design of Ships", Bull. Of Collage of Eng., Basra University, Vol.2, No.1, March, (Iraq-1977), Shama, M. A.,
- 6- "On the Rationalization of ship Structural Design", Schiff und Hafen, March, (Germany-1979), Shama, M. A.
- 7- "ON the Economics of Safety Assurance" Dept. of Naval Architecture and Ocean Engineering, Glasgow University, (UK-1979) Shama, M. A.,
- 8- "CADSUCS, the Creative CASD for the Concept Design of Container Ships", AEJ, Dec. (Egypt-1995), Shama, M. A., Eliraki, A. M. Leheta, H. W. and Hafez, K. A.,
- 9- "On the CASD of Container Ship; State of the Art", AEJ, Dec., (Egypt-1995) Shama, M. A., Eliraki, A. M. Leheta, H. W. and Hafez, K. A.,
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STRUCTURAL and ECONOMICAL CONSEQUENCES
OF SHIP DEFLECTION

BY

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1- SUMMARY:-

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The main causes of ship hull girder deflection are identified and examined. An iterative method is given for calculating the approximate deflection curve resulting from ship loading. The method takes into account the variation in buoyancy distribution along ship length resulting from the deflected shape of the hull girder.

The effects of hull girder deflection on the magnitude and distribution of the shear force and bending moment, along the ship length, are examined. The loss in deadweight resulting from a sagging deflection is identified and the necessary measures to obviate this loss are suggested. The economical consequences resulting from the loss in deadweight are indicated.

It is concluded that:

- i) using high strength steels, structural optimization procedures, increasing ship length/depth ratio, reduction in corrosion allowance and designing to higher working stresses may have an adverse effect on hull flexibility.
- ii) Ship profitability could be marginally improved by reducing hull deflection.

2- INTRODUCTION:-

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The building of the three dimensional complex structure of a ship, over a slip-way or in a building dock, may cause the hull girder, as well as some local areas, to exhibit a deflected form. This built-in deflection, which results basically from welding operations, lack of fit of

fabricated units, settlement of building dock or ground during building or due to the flexibility of the hull girder, cannot be easily controlled or even predicted. However, it could be assumed that, for large ships (above 100,000 tons deadweight), the built-in deflection, over the ship length, is of the order of 100 mm, and normally, it is a sagging deflection.

On top of this built-in deflection, a ship also exhibits an additional deflection due to loading. The hull girder could be idealized by a floating free-free beam of variable cross-section, loaded by the various weights of steel hull, outfittings, machinery, cargo ... etc., and supported by buoyancy forces (1). Under this condition, the hull girder deflects in the longitudinal vertical plane, either in a sagging or a hogging position. The bottom structure also deflects inwards or outwards depending on the local net loading. The magnitude of the hull girder deflection, or local deflection, depends on the distribution of weights and pressures as well as on the variation of shear and bending stiffnesses along the ship length and also along and across the bottom structure. The accuracy of the calculated deflection depends, among other factors, on how the structure is idealized for computation.

Further local and general hull girder deflections result from temperature variations along the ship length and across her breadth and depth. Under certain conditions, this type of deflection may become rather significant and therefore should not be ignored when calculating the total deflection curve of a ship (1);

In the preliminary design stages, it is not necessary to include the effect of the deformed shape of the hull

girder into the subsequent design calculations, whether for hydrostatic, hydrodynamic, structural or operational purposes. However, the importance of calculating the correct deflection curve is realised in the operational problems for ships having wide hatch openings, for large ships working in shallow water zones or passing through shallow water canals, and for long ships having large length/depth ratio.

For these types of ships, the structural and operational problems which may result from excessive hull girder deflection should be taken into consideration. The calculation of the true deflection curve of a ship is also very useful when designing the Block arrangement for docking. Here the problem is very complicated as the true deflection surface depends not only on the load distribution and the flexibility of the hull girder, but also on the flexibility of the building dock, or floating dock, as well as the docking blocks(2). The basic solution of this problem is based on the solution of a beam on an elastic foundation, as given in detail in reference(3).

This paper gives an approximate method for calculating the deflection curve of a ship, using the distribution of weights and buoyancy as well as the variation of shear and bending stiffnesses along the ship length. The shear deflection is taken into account and the total deflection curve is computed with reference to a line joining the two ends of the hull girder as well as to the still water surface. In this analysis, it is assumed that the deflection of the bottom structure between bulkheads, although may reach values higher than 30 mm (4), is relatively small in comparison with the longitudinal vertical deflection of the hull girder, which may exceed 400 mm for large ships.

The effect of hull girder deflection on the distribution of shear force and bending moment, along the ship length, is examined. Two numerical examples are considered for this purpose; a box-shaped vessel and an oil tanker. For the tanker, the distribution of shear force and bending moment, along the ship length, for one loading condition, is given before and after being corrected for ship deflection. The variation in shear deflection between side shell and longitudinal bulkheads are considered elsewhere(5).

Apart from the structural strength consideration, the deflection curve of a ship may also have a significant effect on the load carrying capacity of large ships(6). The load-line mark for a ship in a sagging condition touches the still water surface only at the midship region and not at the ends. The lost buoyancy is in fact equivalent to the reduction in the load carrying capacity of the ship. This reduction in deadweight may reach, in some cases, over 1000 tons. This lost deadweight represents a lost income which may reach a significant figure over the ships service life.

No attempt is made here to investigate the effect of hull deflection on the calculation of hydrostatic, hydrodynamic characteristics or on ship performance and ship operation. Problems associated with shaft alignment and deflection of engine room double bottom (7) is of local nature and is outside the scope of this paper.

The effect of hull flexibility on the dynamic response of ships, with particular reference to slamming has been studied in detail elsewhere(8). Whipping stresses resulting from hull flexibility has been examined in reference(9).

3- DEFLECTION CURVE OF A FREE-FREE BEAM

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The general deflection theory of beams is given in detail in several text books (10). Only a brief summary of this theory, as applied to the case of a floating free-free beam of variable cross-section, is given here.

The total deflection curve of a floating free-free beam, of variable cross-section, under the action of any arbitrary loading system, is composed of two parts, bending deflection and shear deflection.

The bending deflection is calculated from the following general differential equation:

$$\frac{d^2}{dx^2} (EI_x \cdot \frac{d^2 w}{dx^2}) = q_x \quad (1)$$

The shear deflection is calculated from the following equation:

$$w_s = \int_0^x \frac{\lambda F}{GA} dx \quad (2)$$

where: $q_x = f(x) - p(x)$, see fig. (1)

$f(x)$ = downward forces

$p(x)$ = upward supporting forces

λ = a constant depending on the geometry of the cross-section and is given by:

$$\lambda = \frac{1}{AF^2} \int_0^A \tau^2 \cdot dA$$

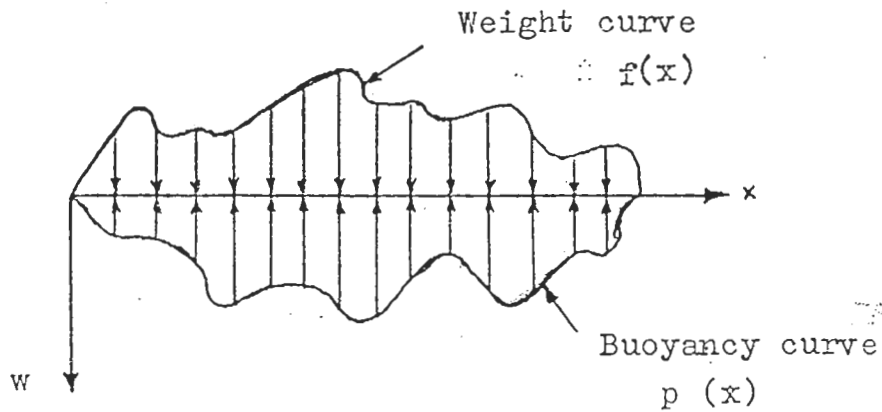


FIG. 1 LOAD DIAGRAM

The total curvature of the elastic line at any position x along the ship length is therefore given by:

$$\left(\frac{d^2w}{dx^2}\right)_t = \left(\frac{d^2w}{dx^2}\right)_b + \left(\frac{d^2w}{dx^2}\right)_s \quad (3)$$

Thus,
$$\left(\frac{d^2w}{dx^2}\right)_t = \frac{M'_x}{EI_x} \quad (4)$$

where: M'_x = bending moment corrected for shear effect

$$= M_x + \alpha E \cdot I_x \cdot q_x \quad (5)$$

$$M_x = \int_0^x \int_0^x q_x \cdot dx^2 \quad (6)$$

$$\alpha = \lambda / AG$$

The equation to the elastic line is therefore given by:

$$(w_t)_x = \int_0^x \int_0^x \frac{M_x'}{EI_x} \cdot dx^2 + \theta_0 x + w_0 \quad (7)$$

where: θ_0 and w_0 are arbitrary constants depending on the chosen reference line and the end conditions, see fig. (2)

For a reference line passing through the two ends of the hull girder, at the still water surface, the two constants θ_0 and w_0 are determined from the following conditions:

$$(w_t)_0 = (w_t)_L = 0$$

The equation to the elastic line is therefore given by:

$$(w_t)_x = \frac{1}{E} \left[\int_0^x \int_0^x \frac{M_x'}{I_x} \cdot dx^2 - \frac{x}{L} \cdot \int_0^L \int_0^L \frac{M_x'}{I_x} \cdot dx^2 \right] \quad (8)$$

On the other hand, if the still water surface is assumed to be the reference plane, see Fig. (2), the two constants θ_0 and w_0 are determined from the following equilibrium

conditions:

$$i. \int_0^L (\Delta q).dx = 0 \quad (a)$$

$$ii. \int_0^L (\Delta q).x . dx = 0 \quad (b)$$

where: Δq = change in load due to hull girder deflection
 $= \gamma \cdot y_x \cdot (w_t)_x$

y_x = breadth of the ship at the water-line and is assumed to be constant over the ship deflection range.

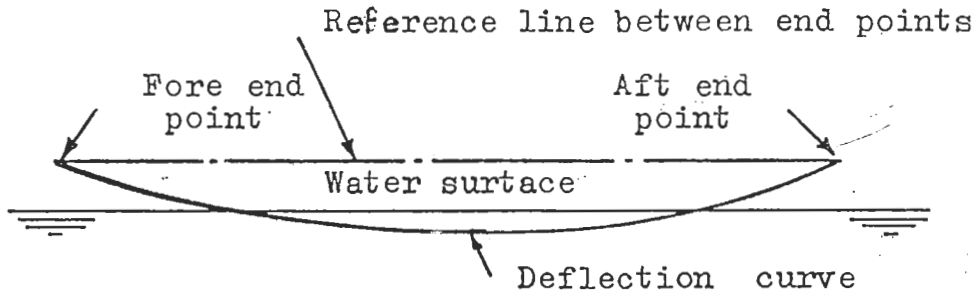


FIG. 2 SHAPE OF DEFLECTION CURVE

Substituting $(w_t)_x$ from equation (7) into (a) and (b), we get:

$$\int_0^L \gamma \cdot y_x \cdot \left[\frac{1}{E} \cdot \int_0^x \int_0^x \frac{M'_x}{I_x} dx^2 + \theta_0 \cdot x + w_0 \right] \cdot dx = 0 \quad (9)$$

$$\int_0^L \gamma \cdot y_x \cdot \left[\frac{1}{E} \cdot \int_0^x \int_0^x \frac{M'_x}{I_x} dx^2 + \theta_0 \cdot x + w_0 \right] \cdot x \cdot dx = 0 \quad (10)$$

Solving equations (9) and (10) for θ_0 and w_0 and then substituting in equation (7), the deflection curve of the elastic line of the hull girder could be determined with reference to the still water surface.

It should be realised that for ship type structures, these calculations can be easily carried out numerically, once the distributions of loading and stiffness along the ship length are known.

4- SHEAR FORCE AND BENDING MOMENT CORRECTION DUE TO SHIP DEFLECTION.

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The deflection of a hull girder alters the assumed distribution of buoyancy. This variation in buoyancy affects the magnitude and distribution of the shearing force and bending moment as well as the shape of the deflection curve.

The calculation of the "correct" deflection curve, as well as the distribution of shearing force and bending moment, could be carried out by an iterative process. The mathematical procedure is given by:

$$q(x) = q_x + \sum_{i=1}^n \delta q_{xi}$$

$$F(x) = F_x + \sum_{i=1}^n \delta F_{xi}$$

$$M(x) = M_x + \sum_{i=1}^n \delta M_{xi}$$

$$w_t(x) = (w_t)_x + \sum_{i=1}^n \delta(w_t)_{xi}$$

where: $i=1,2,\dots,n$ is the number of iterations,

$$\text{and } \delta q_x = \gamma \cdot y_x \cdot (w_t)_x$$

$$\delta F_x = \int_0^x \delta q_x \cdot dx$$

$$\delta M_x = \int_0^x \int_0^x \delta q_x \cdot dx^2$$

$$\delta(w_t)_x = \int_0^x \int_0^x \frac{\delta M_x}{EI_x} \cdot dx^2 + \theta_0 \cdot x + w_0$$

$$F_x = \int_0^x q_x \cdot dx$$

$$M_x = \int_0^x \int_0^x q_x \cdot dx^2$$

This method of calculation has been programmed in Fortran II for the University of Alexandria computer and a flow diagram is shown in Fig. (3)

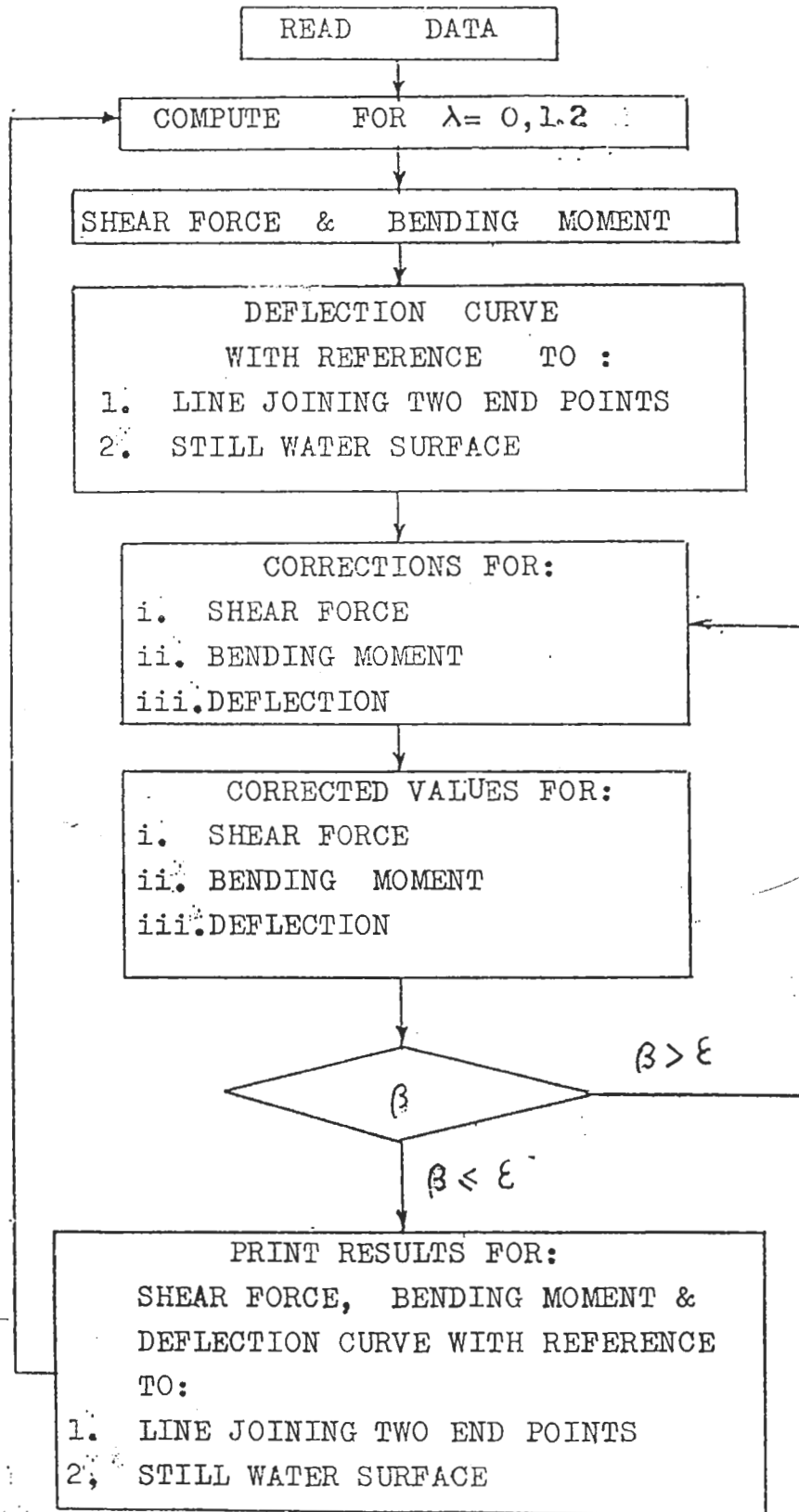


Fig. (3) Flow chart of computer program

The following two cases have been studied:

- i. A box-shaped vessel, see fig. (4)

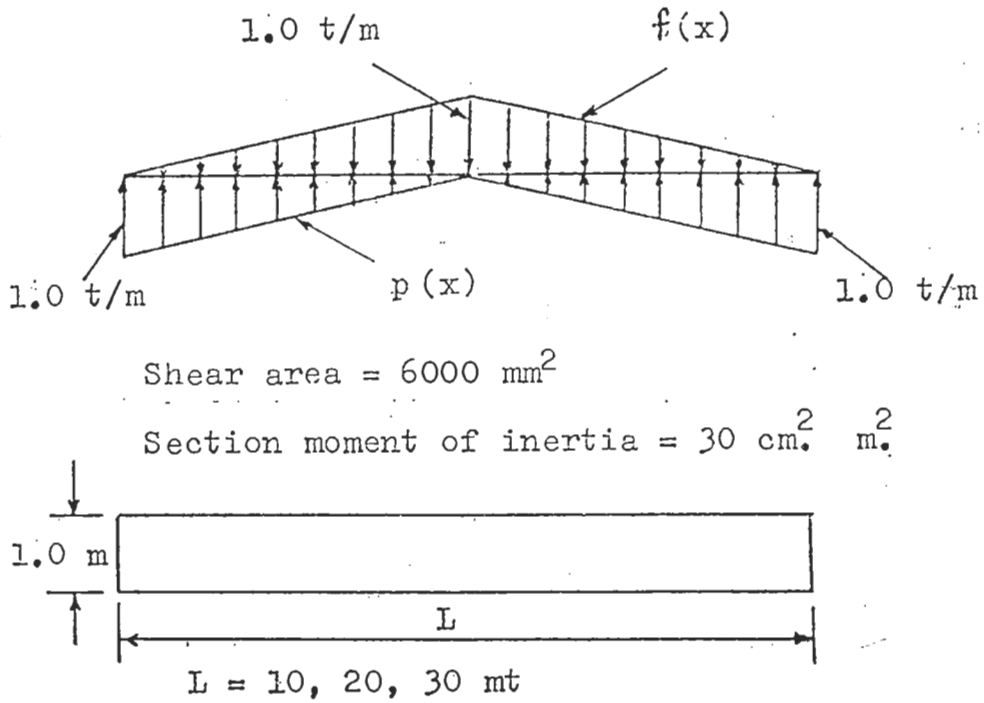


FIG. 4 BOX SHAPED VESSEL

ii. An oil tanker having the following particulars:

$$\text{LOA} = 194.0 \text{ m}$$

$$\text{LBP} = 184.5 \text{ m}$$

$$B_m = 25.6 \text{ m}$$

$$D_m = 14.0 \text{ m}$$

Various conditions of loading and several distributions, along ship length, of shear area and sectional moment of inertia, for both cases (i) and (ii) are studied. The deflection curve is calculated with reference to a line joining the two ends of the idealized hull girder and also with reference to the still water surface, see fig (2). It is assumed in these calculations that $\lambda = 1.2$. The results of one particular loading condition, for the oil tanker, are given in tables (1), and (2).

However, the correction of shearing force and bending moment due to ship deflection could be approximately estimated by assuming that the deflection curve is a second degree parabola, see Appendix (1). This gives for a box-shaped vessel:

$$\delta F_x = \gamma B \Delta \left\{ \frac{4x^3}{3L^2} - \frac{2x^2}{L} + \frac{2x}{3} \right\} \quad (11)$$

$$\delta M_x = \gamma B \Delta \left\{ \frac{x^4}{3L^2} - \frac{2x^3}{3L} + \frac{x^2}{3} \right\} \quad (12)$$

The maximum value of δM_x , assumed amidships, is therefore given by:

$$\delta M_{\max} = C \cdot \gamma \cdot B L^2 \Delta \quad (13)$$

where: C = coefficient depending C_b

For a box-shaped vessel, $C = \frac{1}{48}$

For a diamond-shaped vessel, $C = \frac{1}{360}$

For ships, C varies between $\frac{1}{120}$ to $\frac{1}{60}$ depending on C_b

5- LOSS OF DEADWEIGHT AND SHIP PROFITABILITY
DUE TO SHIP DEFLECTION

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The deflected shape of a ship in a sagging condition allows the loadline marks to touch the still water surface only in the midship region, see Fig. (5) As a result, the ship will not carry its full load. The reduction in deadweight is in fact equivalent to the loss of buoyancy, which is shown in Fig.(5).

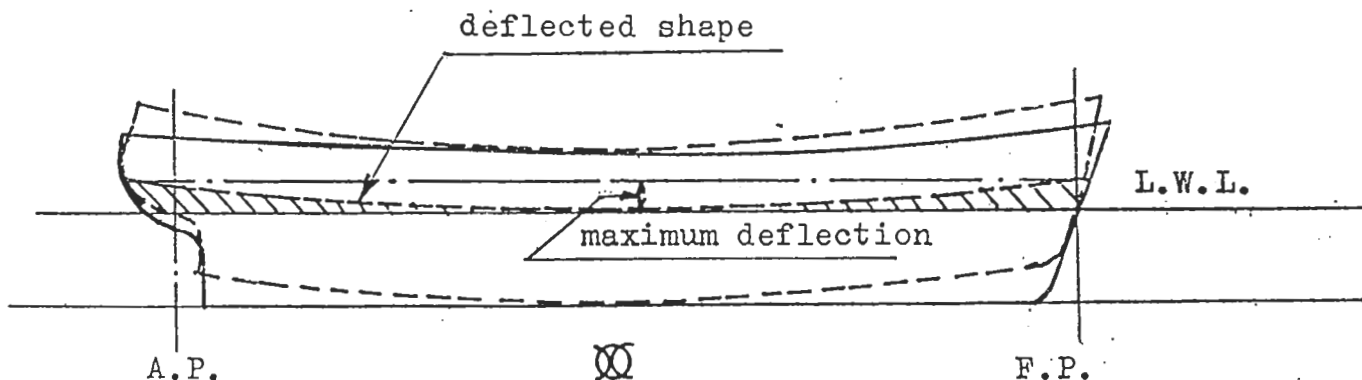


FIG. (5) LOST BUOYANCY DUE TO SHIP DEFLECTION

This loss in buoyancy could be estimated by assuming the deflection curve of a ship to be a second degree parabola. This will give:

$$\begin{aligned} \text{loss in buoyancy} &= \Delta \cdot \left[A_w - \frac{4 \cdot I_L}{L^2} \right] \\ &\approx \Delta \cdot A_w \cdot \left[1 - \frac{0.24}{C_w} \right] \end{aligned} \quad (14)$$

where : A_w = waterplane area

C_w = waterplane coefficient

I_L = moment of inertia of waterplane about amidships

Δ = maximum deflection amidships

For the oil tanker under consideration, this loss of buoyancy is of the order of 700 tons and for larger ships it may exceed 1000 tons.

In order to appreciate the magnitude of this loss in deadweight, the following two examples are given:

a- A general cargo ship.

Assuming a general cargo ship 160 m long, making 15 round trips/year. If the total deflection amidships is about 25 cm, the lost deadweight is approximately 800 tons/round trip. This gives 12000 tons/year and 300,000 tons over 25 years (ships' life).

b- An oil tanker.

For an oil tanker, 250 m long, making 10 round trips/year, the lost deadweight, for a deflection of 30 cm., is about 1600 tons/round trip. This gives 16000 tons/year and 320,000 tons over 20 years, (ships' life). These figures may be much higher for ships having high B/D ratio, such as the modern shallow draught ships.

In order to illustrate the economical consequences of the expected loss in deadweight, the present worth and future amount in L.E. is estimated using the methods given by Benford in (11).

The Present Worth, P, is given by:

$$P = \left[\text{UPWF} \right]_n^i \cdot R$$

where, R = lost income/year
P = present worth of lost income
i = rate of interest
n = ship's life, years

On the other hand, the future amount, S, is given by:

$$S = \left[\text{UCAF} \right]_n^i \cdot R$$

where, UCAF = uniform compound amount factor

Assuming F = freight rate in L.E./ton, we have:

a- Cargo ship:

lost income/round trip = L.E. 800 F

Hence, $R = \text{L.E. } 12,000 \text{ F}$

The Present Worth, P , of lost income, for two values of i , are given by:

i	$P, (\text{L.E.})$
10%	108,800 F
6 %	153,500 F

The future amount, S , of lost income, for the same two values of i , are given by:

i	$S, (\text{L.E.})$
10%	1,177,000 F
6%	659,500 F

b- Oil tanker:

lost income/round trip = L.E. 1600 F

Hence, $R = \text{L.E. } 16,000 \text{ F}$

The Present Worth, P , of lost income, for the same two values of i , are given by:

i	P, (L.E.)
10%	136,200 F
6%	183,500 F

The future amount, S, of lost income, for the same two values of i, are given by:

i	S, (L.E.)
10%	914,500 F
6%	588,500 F

It should be realised that the figures for the cargo ship are obviously on the high side. This is because these figures are based on the assumption that the ship is fully loaded all the time, which is not always the case for a general cargo ship. On the other hand, the figures for the oil tanker are more realistic since a tanker is either fully loaded or ballasted.

These figures, however, are based on a constant freight rate and therefore give pessimistic estimates of the expected loss in income. With escalating freight rates, and larger deflections, the above values of P and S will certainly be much higher. This expected loss in income should, therefore, justify all the feasible measures necessary to obviate, or reduce, excessive hull deflections.

In order to avoid this loss of deadweight, the ship could be built originally with a hogging deflection, such that when loaded, the sagging deflection will approximately straighten the ship out. Alternatively, a correction should be given to the load-line marks amidships such that due account is taken of ship deflection. The correction to freeboard is of the order of $\frac{\Delta}{3}$, Δ being the total deflection amidships. The former solution may impose some difficulties during ship construction and may increase the building cost, but the additional income will certainly offset the increased building cost. This is particularly important for ships normally running fully loaded. On the other hand, the second solution requires an International agreement on the scope and limitations of the freeboard correction. This requires a separate investigation and is beyond the scope of this paper.

6- CONCLUDING REMARKS

The main conclusions drawn up from this investigation are summarised as follows:

1. Because of the increased knowledge on local, dynamic and wave induced loading on ships, Classification Societies may raise their allowable stresses with a subsequent reduction in ship sectional modulus and moment of inertia. Also, when effective measures against corrosion are considered, Classification Societies may allow a reduction in corrosion allowance with a consequent reduction in ship sectional moment of inertia.

This increased hull flexibility may be further increased by using a high L/D ratio. Also, increasing strength/

weight ratio by using high strength steels or reducing weight/strength ratio by using structural optimization procedures may cause an appreciable reduction in the stiffness of hull girder. Effective measures should therefore be taken to control all these factors so as not to impair hull girder stiffness.

2. Proper distribution of cargo loading may have a significant effect on hull girder deflection.
3. Ship profitability could be marginally improved by reducing hull deflections, building the ship with a hogging deflection or by reducing the minimum freeboard by $\frac{\Delta}{3}$, Δ being the maximum deflection amidships.
4. Ship deflection may reduce the maximum values of shearing force and bending moment by more than 3% and the end draughts by 2%.
5. Shear deflection may reach up to 20% of total deflection, and therefore can not be ignored.
6. Excessive hull girder deflections may aggravate the operational problems resulting from the sinkage and trim associated with ships passing through shallow water zones.

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LIST OF NOTATION

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A	=	Shear area
A_w	=	Waterplane area
B	=	breadth at waterline
C	=	a coefficient
C_b	=	block coefficient
C_w	=	waterplane area coefficient
E	=	modulus of elasticity
F	=	freight rate
F_o	=	constant depending on the reference line.
F_x	=	shear force at a distance x
G	=	modulus of rigidity
i	=	interest rate
I_L	=	moment of inertia of waterplane about amidships.
I_x	=	second moment of area of the longitudinal material for a floating free-free beam, or ship section, at a distance x.
L	=	ship length
M_x	=	bending moment at a distance x
M'_x	=	bending moment corrected for shear action
M_o	=	constant depending on the reference line
P	=	present worth
q_x	=	load at any position x along the ship length (or a floating free-free beam)
S	=	lost future income
UCAF	=	uniform compound amount factor
UPWF	=	uniform present worth factor
w_s	=	shear deflection
w_b	=	bending deflection

- w_0 = a constant of integration
- w_t = total deflection ($w_t = w_s + w_b$)
- x = distance along the ship length from the aft end
- y_x = breadth of the ship at the water-line and at a distance x
- α = a factor associated with shear stiffness ($\alpha = \lambda/AG$)
- β = computed difference for either deflection or bending moment during the iteration process
- γ = density of water
- θ_0 = a constant of integration
- λ = a constant depending on the geometry of the cross-section
- τ = shear stress
- ϵ = a pre-determined value representing the acceptable margin to be used in the iteration process
- Δ = maximum deflection amidships relative to a line joining the two-ends of the idealized hull girder.
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APPENDIX (1)

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Correction term of maximum bending moment.

Assuming that the deflection curve is a second degree parabola given by:

$$(w_t)_x = ax^2 + bx + c \quad (A1)$$

where: a, b and c are constants to be determined from the end conditions, see Fig. (6) :

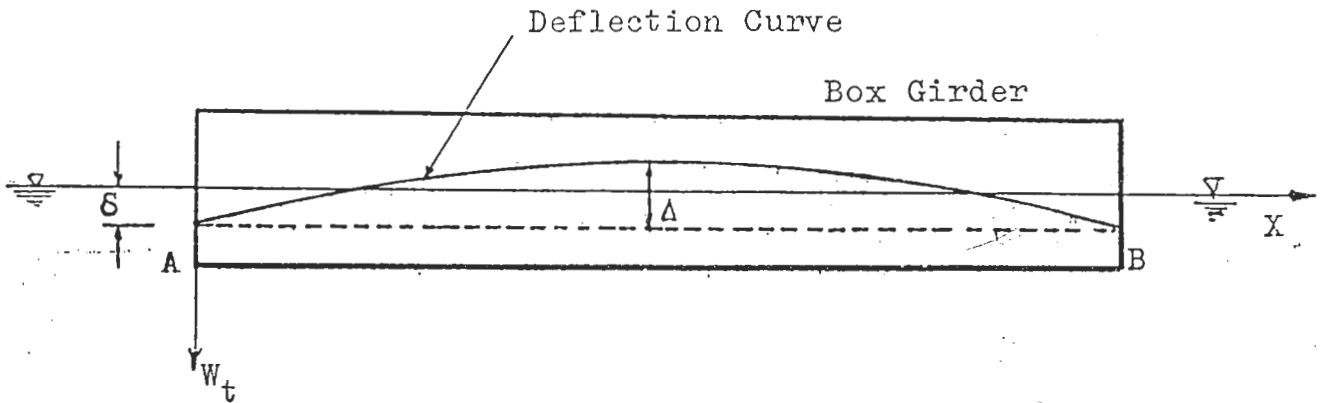


FIG. (6) ASSUMED DEFLECTION CURVE

$$\text{at } x = 0, \quad \delta(w_t) = \delta$$

$$\text{at } x = \frac{L}{2}, \quad \delta(w_t) = -(\Delta - \delta)$$

$$\frac{d \delta(w_t)}{dx} = 0$$

Substituting in (A1), we get:

$$\delta(w_t)_x = 4 \Delta \left\{ \left(\frac{x}{L} \right)^2 - \frac{x}{L} \right\} + \delta \quad (A2)$$

The load intensity/unit length is given by:

$$\begin{aligned} q_x &= y_x \cdot \delta(w_t)_x \cdot \gamma \\ &= y_x \cdot \gamma \cdot \left\{ 4 \Delta \left[\left(\frac{x}{L} \right)^2 - \frac{x}{L} \right] + \delta \right\} \end{aligned} \quad (A3)$$

The shearing force and bending moment distributions are therefore given by:

$$\begin{aligned} \delta F_x &= \int_0^x \delta q_x \cdot dx \\ &= \int_0^x y_x \left\{ 4 \Delta \cdot \left[\left(\frac{x}{L} \right)^2 - \frac{x}{L} \right] + \delta \right\} \cdot dx + C_1 \end{aligned} \quad (A4)$$

$$\text{and } \delta M_x = \gamma \cdot \int_0^x \int_0^x y_x \cdot \left\{ 4\Delta \cdot \left[\left(\frac{x}{L} \right)^2 - \frac{x}{L} \right] + \delta \right\} \cdot dx^2 + C_1 x + C_2 \quad (\text{A5})$$

The end conditions give:

$$C_1 = C_2 = 0$$

For a box-shaped vessel, the relationship between δ and Δ could be obtained from the condition:

$$\delta F_x = 0 \text{ at } x = \frac{L}{2}$$

$$\text{Hence, } \delta F_x = \gamma B \Delta \cdot \left[\frac{4x^3}{3L^2} - \frac{2x^2}{L} + \frac{2x}{3} \right] \quad (\text{A6})$$

$$\text{and } \delta M_x = \gamma B \Delta \cdot \left[\frac{x^4}{3L^2} - \frac{2x^3}{3L} + \frac{x^2}{3} \right] \quad (\text{A7})$$

For a diamond shaped vessel,

$$\delta = \frac{5}{6} \Delta$$

$$\text{Hence, } \delta F_x = \gamma B \Delta \cdot \left[\frac{x^4}{L^3} - \frac{4x^3}{3L^2} + \frac{5}{12} \cdot \frac{x^2}{L} \right] \quad (\text{A8})$$

$$\text{and } \delta M_x = \gamma B \Delta \cdot \left[\frac{x^5}{5L^3} - \frac{x^4}{3L^2} + \frac{5}{36} \cdot \frac{x^3}{L} \right] \quad (\text{A9})$$

The maximum value of δM_x occurs at amidships and is given by:

$$\delta M_x = C \gamma BL^2 \Delta$$

where C is a coefficient given by:

For full ships $C = \frac{1}{60} - \frac{1}{70}$

For fine ships $C = \frac{1}{100} - \frac{1}{120}$

TABLE (1)

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Shear Force And Bending Moment
Distributions

STN.	Shear Force x 10 ⁻³ tons		Bending Moment x 10 ⁻⁴ t.m.	
	Before correction	After correction	Before correction	After correction
1	0.00	0.00	0.00	0.00
2	- 0.376	- 0.369	0.167	0.165
3	- 1.121	- 1.105	0.844	0.832
4	- 2.050	- 2.027	2.287	2.256
5	- 2.933	- 2.905	4.559	4.504
6	- 3.401	- 3.371	7.449	7.367
7	- 3.021	- 2.993	10.373	10.264
8	- 2.428	- 2.405	12.842	12.710
9	- 1.623	- 1.607	14.659	14.509
10	- 0.652	- 0.645	15.651	15.490
11	0.355	0.352	15.724	15.561
12	1.317	1.305	14.882	14.725
13	2.094	2.074	13.232	13.090
14	2.567	2.540	10.999	10.879
15	2.744	2.714	8.459	8.364
16	2.553	2.523	5.919	5.851
17	2.038	2.012	3.699	3.657
18	1.432	1.412	1.989	1.968
19	0.825	0.814	0.832	0.825
20	0.219	0.216	0.229	0.228
21	0.00	0.00	0.00	0.00

- ve downwards shearing force and hogging moment
+ ve upwards shearing force and sagging moment

TABLE(2)

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Deflection Curve

STN.	Deflection Curve(mm)			
	With reference to W.L.		With reference to end points	
	corrected	uncorrected	corrected	uncorrected
1	-181.6	-161.3	0.0	0.0
2	-146.0	-127.1	38.9	40.3
3	-110.1	- 92.6	78.0	80.8
4	- 74.0	- 57.9	117.4	121.5
5	- 38.3	- 23.6	156.5	161.9
6	- 3.8	9.3	194.2	200.8
7	27.7	39.1	229.0	236.7
8	54.5	64.0	259.1	267.6
9	74.7	82.0	282.6	291.6
10	86.4	91.3	297.6	307.0
11	88.5	90.6	302.9	312.3
12	85.0	84.3	302.8	312.0
13	76.0	72.1	297.0	306.0
14	56.6	49.4	281.0	289.3
15	28.1	17.3	255.7	263.2
16	08.1	- 22.7	222.8	229.3
17	- 50.2	- 68.6	184.0	189.4
18	- 96.3	- 118.8	141.1	145.3
19	- 145.2	- 171.7	95.6	98.4
20	- 195.8	- 226.4	48.3	49.7
21	- 247.4	- 282.2	0.0	0.0

- ve above water surface

+ ve below water surface